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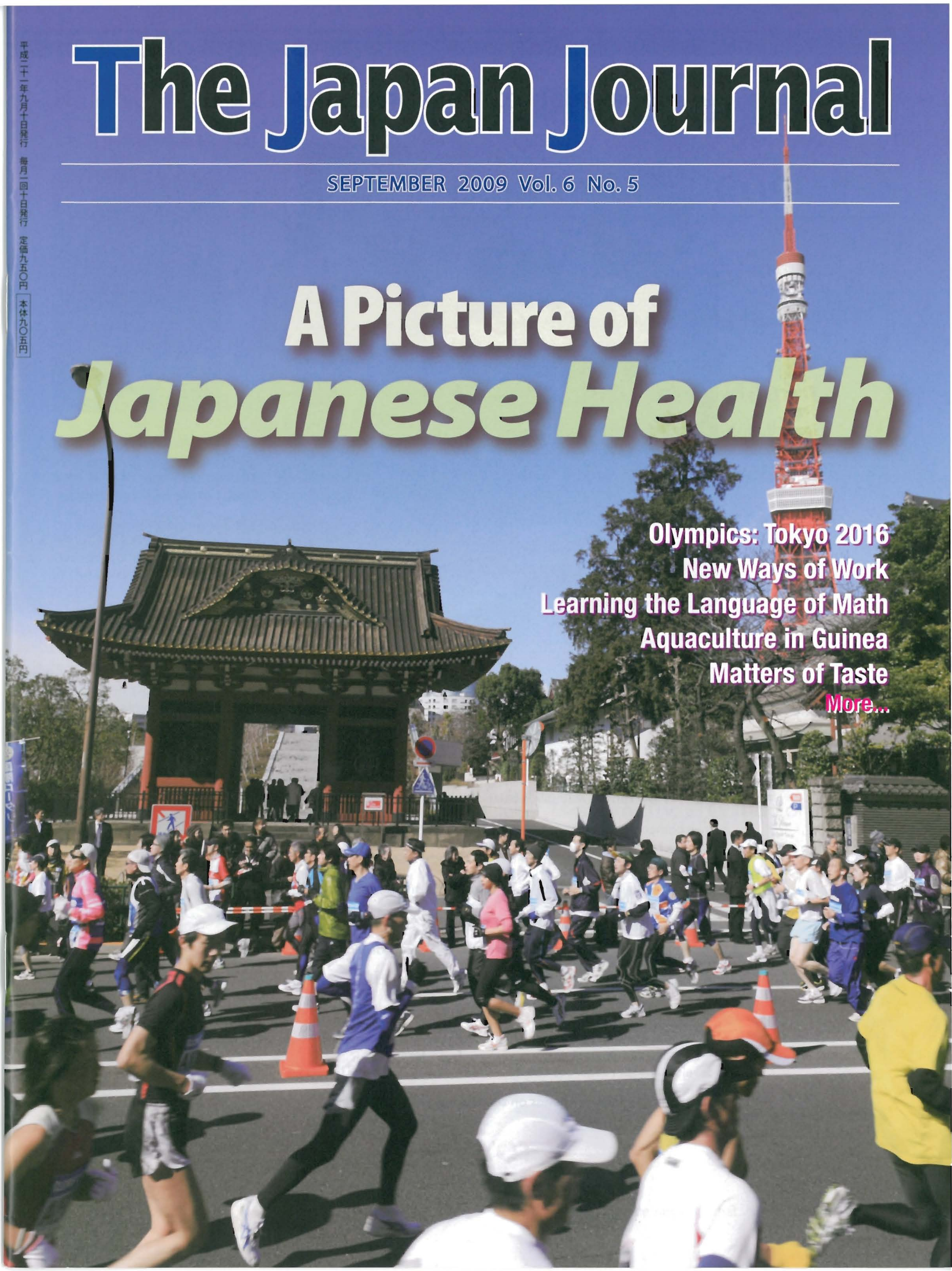
A Picture of *Japanese Health*



Strength in Numbers

Established in 2007, the Alliance for Breakthrough between Mathematics and Sciences supports research activities in mathematical science that are motivated by social needs. Research Director **Nishiura Yasumasa** explains the "math" behind the program and highlights ways in which the projects undertaken might influence our daily lives.

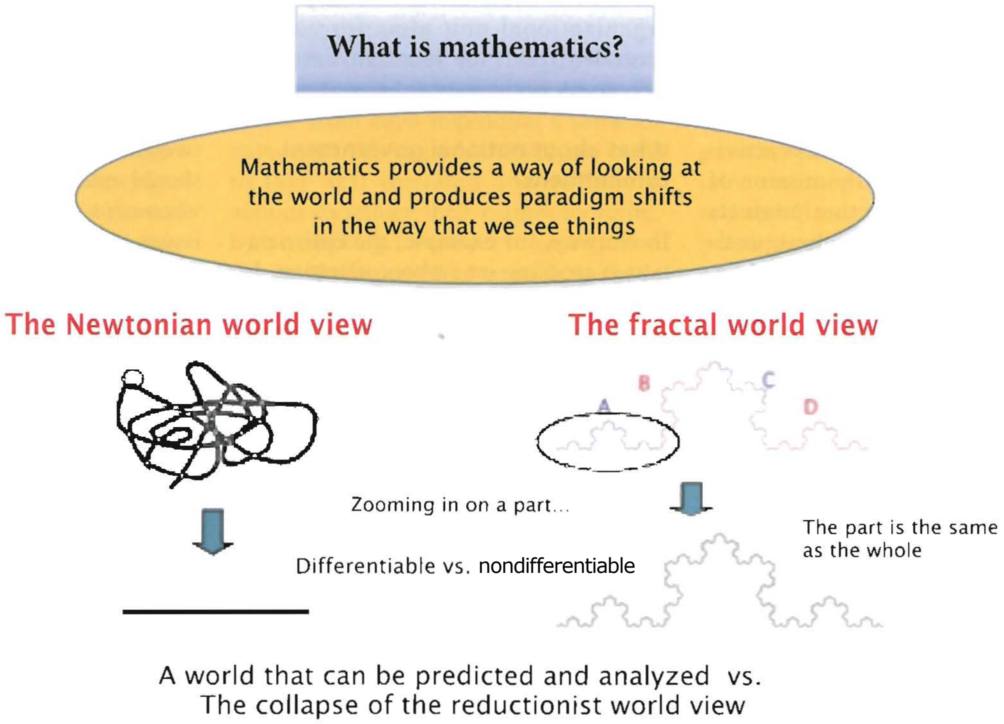
Mathematics has always produced ways of looking at the world that are ahead of their times. From the Ptolemaic worldview of the harmony of the spheres that accompanied astrology, through to the Newtonian worldview, all the way down to the modern chaos/fractal view of nature, mathematics has brought about a series of paradigm shifts epitomized by the Copernican revolution. These shifts can be quite profound, because if we zoom in on something in the Newtonian depiction of reality, a linear approximation can always be found for any object, no matter how complicated, but from a fractal perspective there is always a miniature version of the whole no matter how far we zoom in, so that we are presented with a world where it is impossible to separate parts from the whole, or the self from the non-self (see **figure**).

However, the true greatness of mathematics lies in the way that it uses "invisible concepts" to describe "the visible world" (that is, in the way that mathematics approaches the problem of how to apprehend the universe and the natural world around us). A simple example of this is the concept of "infinity." In fact, the Newtonian representation of reality describes and predicts the visible, macroscopic motions of the planets using the invisible concept of "secondary derivative," which in turn is supported by the invisible concepts of "infinitesimal" and "infinite." Without mathematics it would never have been possible to represent leaves, coastlines or even whole landscapes using infinitely recursive fractals. However, it is difficult to perceive this infinite recursion directly. These endeavors are not driven by a desire simply to reduce the beauty of nature to a meaningless mathematical description, but rather by a desire to achieve a deeper understanding of its mystery.

Invisible Technology

However, since it is invisible, many of the fruits of mathematics are used in

Representations of the World that Mathematics Has Produced



ways that are invisible from the standpoint of the sciences, which tend to regard mathematics as simply a "tool." In fact, mathematical logic is used in ways that are truly invisible as part of our everyday life, such as in mobile telephones, as well as in more obvious applications such as barcodes, encryption logic and non-invasive measurement techniques, such as fMRI. Calculating orbits to minimize the fuel consumption of space probes for interplanetary exploration is an interesting example of a practical application of the principles of dynamics, but this too has only just been taken up by the newspapers, as astronauts have been in the news. It is certainly not generally known that mathematics is hidden all around us, from GPS car navigation systems through to special effects in the movies. From the standpoint that the highest echelon of technology is to be "invisible technology," mathematics can surely be said to have well and truly secured that position.

However, mathematics' more intrinsic role does not reside in its aspect as a "tool" or "hidden supporter of technology" but rather in a completely different function. That function is to provide a point of view for identifying the essence of things that we have no idea how to understand, and a framework and a perspective for thinking about such things.

Human beings are living creatures and, as such, have a tendency to disregard things that are not important to themselves and things that have no meaning to themselves. "Looking but not seeing," or—this is the fate of all living creatures that have resulted from evolution—"seeing things as they should be" are things that we often experience in the form of optical illusions. There are many examples in history of things that no one has bothered paying attention to, or things that have been regarded until quite recently as "errors" (or as "incomprehensible" or "noise") suddenly being rescued from obscurity and becoming the focus of attention as soon as it is discovered that they contain an embedded mathematical object with a certain structure.

In terms of the internal development of science, it is certainly not a coincidence that the logical frameworks that allowed the development of quantum mechanics and relativity theory early in the twentieth century had been developing independently within mathematics. This kind of mutual exchange of perspectives for looking at the world expands even further, so that things that were once invisible start to become apparent. This is especially true for things that are difficult to imagine with our normal human perceptions. The fact that coming up with a good, universal "definition" takes a huge amount of time and effort by many bright people is indicative of just how difficult it is to depart from our intuitive depiction of reality.

A Non-Physical Vantage Point and Current Uncertainty

However, the importance of the perspective whereby the invisible becomes visible as a result of putting on our "mathematical glasses" is not always properly appreciated by researchers in the other sciences. This is because they tend to persist with an approach that starts with "things" where these "things" cause certain effects. For them, ideas that are removed from real "entities" such as "matter" or "life" are nothing more than empty theory, with no ability to explain reality. However, this "reality" is somewhat shifty. That the things that we perceive with our senses or measure with our equipment cannot be simply called "reality" or "entities" is abundantly clear from the history of the development of measurement techniques and their interpretations. The reality that blind people construct without a sense of sight, using only other senses such as touch and hearing, is different to the reality of an able-bodied person, but it still forms a rich world of its own. After all, we have developed our sensory nervous systems through an expedient process of evolution in order to survive on this planet. This is extraordinary, but it also has its limitations.

On the other hand, there are also "social entities," which are different from physical entities. Good examples are currency and interpersonal relations. However, it is difficult to see some things that have even larger impacts. Why is it that the world is so uncertain and unstable, where anything could happen? That is because the agents that impose restrictions on the way that we live our lives were, until modern times, either natural forces that are difficult to control, such as the sun, moon and natural disasters or visible things like monarchs and other rulers, but now this agency has moved to "social mechanisms," which are both invisible and ubiquitous.

These non-physical, abstract structures are where mathematics really comes into its own. In fact, perhaps there would be no hope of finding fundamental solutions to these kinds of problems without the involvement of mathematics. There are many areas—from global warming to economic fluctuations, psychology and risk management—where we have still only partially worked out the big picture and elucidated the mechanisms involved, but what all of these areas have in common is that there are a huge number of contributing factors and it is difficult to reach an understanding using a simple cause-and-effect framework (see **table** p. 30). However, these changes are not completely random either. It may be difficult to make precise predictions, but in some areas it has become possible to work out macroscopic trends within a certain margin of error. In the 2007 IPCC report on global warming, different researchers arrived at almost identical conclusions (within a certain range) despite the fact that they were using different mathematical models. This demonstrates how the language of mathematics is essential for overcoming our self-interest in order to stand on common ground.

Aiming for "Connective Knowledge"

If we know the direction in which we should proceed, then by working out the details we can expect a return proportional to the time and effort invested, and in fact research on these kinds of projects is endorsed just about everywhere. However, there are many problems that cannot be solved simply by working out details or by thorough enumeration. This mass of data is essential,

The Origin of Uncertainty

Where does uncertainty come from in the modern world?

* A dramatic increase and collision of human desires on a limited planet

* The agents that place restrictions on our way of living have shifted from visible things to invisible things.
  - Global warming, economic fluctuation, risk management, psychological uncertainty, ..

* The limitations of a simple "cause-and-effect" framework
  - Mutual interdependencies, feedback loops and multiple cause-and-effect relationships
  - Perpetrators and victims are either the same, or difficult to distinguish

* Dissociation of our awareness of the scale of time and space
  - We have difficulty recognizing extremely slow changes
  - Local changes are connected to global changes

* Human beings think linearly, but reality is non-linear
  - We tend to think that current trends will continue, but these trends could collapse at any time

but working out the story that it tells is another problem entirely. Perhaps only mathematics gives people the imagination and the powers of abstraction to overcome our limitations, by integrating, if only partially, science, which has become increasingly bloated even as it becomes ever more fragmented. The reservoir and perspectives of mathematics, which have been built up assiduously in an "invisible form," do not simply provide the sciences with a language to describe the world, they are also a treasure trove which holds the key to the kind of departures described above. When we can't see something no matter how hard we look, we need "glasses" that will enable us to see, and one source of such insights may come from the pursuit of mathematics.

The question is, who will bell the cat, namely, who can attach the label of "a mathematical vantage point" to these "complex structures with a massive amount of data?" This problem lies in the opposite direction to the fragmentation and enlargement of science that was described above, and is a life-and-death question for modern science, including mathematics. This is precisely why it is so urgent that we cultivate talented people who can act as an interface between mathematics and the sciences, with a foot in each camp. We need mathematicians and mathematical scientists who have both a strong intellectual foundation in a certain area of mathematics and also what Ivica Osim, the former coach of the Japanese national soccer team, called "polyvalence." In other words, it is important that we have a "team," a network of sophisticated mathematicians that can make full use of "developed knowledge" and "connective knowledge" in all directions. This does not conflict with the position of mathematicians until now. Rather, by utilizing them as important nodes in the network, both mathematics and the sciences can coexist in harmony. Mathematicians need to become more open, while also becoming aware of the diverse missions with which mathematics has been charged.

Mathematics as a Language of Mutual Understanding

In 2007, the Alliance for Breakthrough between Mathematics and Sciences (www.math.jst.go.jp) was established by the Japan Science and Technology Agency (JST) as part of the Core Research for Evolutional Science and Technology (CREST) program. The first aim of this alliance is to demonstrate the potential of mathematics as a "connective knowledge" in society, by applying the last numerical technology and the voluminous mathematical assets that we have accumulated so far to some of the most difficult problems that we are facing today, as described above. The second aim is for mathematics to become the core component in the formation of a "language of mutual understanding" for all humanity. The Precursory Research for Embryonic Science and Technology (PRESTO) program grants for individual research projects by young researchers started in 2007, followed by the CREST program for team-based research in 2008. Twenty PRESTO research projects and three CREST projects are already underway. These projects address a number of areas in materials science, life sciences, environmental sciences, information and communications, transport, finance and medicine, with projects such as "Mathematical sciences collaborating with clinical medicine," "Unified analysis on various transportations and solution of their traffic congestion," and "Mathematical models of visual perception by means of wavelet frames" being selected for PRESTO grants and "A mathematical challenge to a new phase of material science," "Innovations in controlling hyper redundant and flexible systems inspired by biological locomotion" and "Harmony of Gröbner bases and the modern industrial society" being selected for CREST grants.

Hopefully, the young researchers participating in these programs will go on to become an interface between mathematics and the sciences. Please see the website above for more information about these research projects. In the following section I would like to discuss two ways in which the mathematical perspective and methodology behind these collaborative research projects can influence the way that we go about our daily lives.

1) An awareness of the scale of time and space

We live for about seventy or eighty years, and in our day to day lives our ability to perceive spatial dimensions generally ranges from a few millimeters to a few hundred meters, but in these respects we are not necessarily better than other animals. And in terms of our senses of smell and touch, we do quite poorly. These extremely limited sensory perceptions give us many erroneous impressions, such as the impression that the earth and the people on it have always existed just as they do now, and that they will probably continue to remain the same into the future. However, we now know that initially the earth's atmosphere contained almost no oxygen, and that the current atmosphere was created in a slow process that included the emergence of photosynthetic bacteria, the formation of the ozone layer and the oceans, and the evolution of higher life forms. We also know that the atmosphere will continue to change in the future, and that the current problem of CO_2 emissions is just one part in this greater process.

We humans find it difficult to imagine such extremely slow changes—even though it is just a blink of an eye when viewed in terms of geological time. We are also not very good at coming to grips with feedback mechanisms or extremely long chains of cause and effect.

In Japan there is a proverb that says, "If the wind blows the bucket makers prosper," a metaphor meaning that seemingly insignificant everyday events can have unexpected consequences.

Problems such as environmental problems, energy problems and the food-supply problem tend to be discussed from an economic perspective, focusing on the interdependencies between countries. However, all of these things, whether they be food or fossil fuels, have their origin in energy from the sun, and it is only because this energy is stored in a visible form that it becomes a target of contention. Yet at the same time, we remain unaware of the gift of energy from the sun that comes down to us in other, less visible forms. We need to think about these problems comprehensively, from the perspective of geothermal energy, ocean currents and the universe, and not just wind power or solar energy.

The fundamental principle when it comes to applying mathematical models to reality is to accurately grasp what comes in and out of the system. In the metaphor above, this involves first pinning down "the wind" and "the bucket makers" and then gradually clarifying the network that joins them. The mathematical structure that is derived as a result has the potential to be applied to a wide range of problems.

2) Mutual exchange between the micro and the macro

Traffic jams come about as a result of drivers increasing their speed and closing the gap between them and the vehicle in front to satisfy their desire to arrive at their destination as soon as possible. In other words, traffic jams are caused by people. The solution provided by a mathematical model of traffic jams is "take your time and maintain a safe distance between you and the car in front." This result may take some people by surprise, but the mathematical model gives us a precise answer to the quantitative question of what speed and inter-vehicular distance produces the maximum flow of cars per hour. It is mathematics that tells us how "micro" information about the speed of one car and the distance to the next car is linked to the "macro" information about the overall flow of traffic. Many problems remain, such as the legal problems that have to be solved in order to implement this in practice and the question of how to exchange information between drivers, but basically "macro" information should feedback to drivers in order to give them an incentive to adjust their behavior. There is already some feedback when it comes to fuel consumption, but if drivers were given information about how their driving contributed to an increased traffic flow then it is likely that we would see less people cutting in recklessly. And if drivers observed these mathematical behavioral norms, this would result in a significant reduction in CO_2 emissions.

The Age of Mathematics

As we have seen above, because mathematics focuses on the *relationships* between objects rather than depending on any particular object, it can be expressed in the same form regardless of what the object is. In that sense, we are living in an age where mathematics is rapidly becoming more important. At the same time as individual objects are being divided into their constituent elements and homogenized, changing relationships and exchanges of information are also becoming more rapid and more widespread. In these circumstances, it seems that those who can rapidly read the structure and dynamics of the network world, where connectedness is all important but the actual objects do not matter, and then make fast and accurate predictions will be the winners —Google being a typical example of this trend. However, such circumstances also have an extremely vulnerable aspect, namely that there is also a high risk of collapsing suddenly. In other words, we live in circumstances where what is known in the non-linear sciences as "cooperative phenomena" can occur easily. If this is the case, then all we can do is to cultivate a sensibility that does not overlook global information or signs, while creating and implementing mathematical behavioral norms and ethical norms that combine both the "micro" and the "macro" perspectives. Mathematical science teaches us that it is because these are cooperative phenomena that many small individual contributions become visible as a macroscopic result.

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