

## **Bulk-edge duality for two-dimensional topological insulators**

*Marcello Porta*  
*University of Zürich*

Topological insulators are newly discovered materials, which behave as usual insulators in the bulk but carry spin currents on their edges. The presence of the currents is stable against perturbations which respect time-reversal symmetry. I will discuss a duality between the bulk and edge descriptions of two-dimensional topological insulators, namely the relation between the presence/absence of edge currents and the value of a certain bulk  $\mathbb{Z}_2$  topological invariant. This is joint work with G. M. Graf.

## **Bulk-edge correspondence and the Gysin map in K-theory**

*Shinichiroh Matsuo*

*Osaka University*

## **The FKMM invariant**

*Kiyonori Gomi*  
*Shinshu University*

Atiyah's "Real" vector bundle and its cousin, Dupont's "Symplectic" (or "Quaternionic") vector bundle, respectively construct variants of topological K-theory called KR-theory and Ksp-theory (or KQ-theory).

These K-theories attracts recent interest due to their applications to the classification of topological insulators. The FKMM invariant is an invariant of "Symplectic" vector bundles originally introduced by Furuta, Kametani, Matsue and Minami around 2000. I will talk about recent development about this invariant based on joint works of Giuseppe De Nitts.

## Topological phases and the ABS-construction

*Peter Bouwknegt*

*The Australian National University*

Topological insulators and superconductors are many-fermion systems possessing an unusual band structure that leads to a bulk band gap as well as topologically protected gapless extended surface modes.

It was recently realised that deformation classes of gapped Hamiltonians are naturally classified by K-theory. This classification parallels the classification of the 2 complex and 8 real symmetry classes of Hamiltonians (the '10-fold way') of Altland and Zirnbauer, and naturally leads to a periodic table of topological insulators.

In my first talk I will give a brief overview of these developments from the point of view of the Atiyah-Bott-Shapiro construction of K-theory, and in my second talk I will extend these methods to certain topological phases in strongly interacting systems described by parafermions.

## **Bulk-edge correspondence via coarse geometry**

*Yosuke Kubota*  
*University of Tokyo*

In this talk we introduce a new mathematical approach for the bulk-edge correspondence for non-periodic Hamiltonians. This is a reinterpretation of the noncommutative geometry of the quantum hall effect introduced by Connes and Belisard--van Elst--Schulz-Bardes in terms of coarse geometry.

Here we deal with the crystal itself as a metric space and Hamiltonians are topologically classified by the K-group of the Roe algebra. The coarse Baum--Connes conjecture enables us to compute the K-group itself.

In particular the Kane--Mele  $\mathbb{Z}_2$  invariant is defined for a non-periodic Hamiltonian.

## **Electronic Berry curvature induced by the interaction with magnetostatic waves**

*Koji Sato*

*Tohoku University*

Magnetostatic waves are the collective excitations of magnetic moments in a long wavelength limit, where the interaction is dominated by the magnetic dipolar interaction. One of the magnetostatic modes, known as forward volume mode, has non-trivial Berry curvature, and it exhibits the thermal Hall effect. When electrons are couple with these magnetostatic modes, the electrons can acquire non-trivial Berry curvature leading to anomalous Hall effect.

## **Torus fibrations and localization of index**

*Takahiko Yoshida*

*Meiji University*

We report a recent progress of the joint work with H. Fujita and M. Furuta on an index theory of a Dirac-type operators on possibly non-compact Riemannian manifolds. In our work we make use of a structure of torus fibration on the end, and perturb a Dirac-type operator in terms of first order differential operators along fibers on the end which satisfy a kind of acyclic condition. The perturbation allows an interpretation as an adiabatic limit or an infinite dimensional analogue of Witten deformation. As an application, we show that in the case of Lagrangian fibrations the Riemann-Roch index can be described as the sum of the number of Bohr-Sommerfeld fibers and the contributions from singular fibers.

Joint work with Hajime Fujita and Mikio Furuta.

**Non-commutative Chern numbers,  
KK theory and index computations**

*Alan Carey, The Australian National University*

*Christopher Bourne, University of Wollongong*

Topics to be covered include: Fredholm modules and spectral triples, K-theory and K-homology, Chern character as a map to cyclic theory, The Connes-Chern index pairing, KK definitions and KK pairings, with examples drawn from condensed matter models.



## **Symmetry and topology in insulators and superconductors**

*Masatoshi Sato*  
*Nagoya University*

Recently, there has been much interest in topological phases in condensed matter physics. After the historical review of topological phase, I would like to explain our understanding of relation between symmetry and topology in insulators and superconductors by using some of examples that are indeed realized experimentally.

## **K-theory as obstructions between topological phases**

*Guo Chuan Thiang*  
*University of Adelaide*

Attempts to classify "homotopic Hamiltonians" in an absolute sense face difficulties with respect to compatibility with isomorphism of systems. Parametrisation-dependent definitions may also introduce artificial features into the topological space of allowed Hamiltonians. These are resolved using a model for K-theory which presents it as groups of path-obstructions between symmetry-compatible Hamiltonians. Only the physically meaningful data of quantum mechanical symmetry constraints for free-fermions enters the discussion. They generate a noncommutative twisted crossed product  $C^*$ -superalgebra, whose K-theory is then studied. The collection of symmetry-compatible free-fermion Hamiltonians, up to homotopy, is akin to a torsor for its K-theoretic group of obstructions, clarifying the precise role of K-theory in this subject. A robust version of the dimension-shift phenomenon can be proven using the noncommutative results of Connes and Packer-e Raeburn.

References (if required)

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- [3] D. Freed and G. Moore, Twisted equivariant matter, Ann. Henri. Poincare, Vol. 14, No. 8, pp. 1-97 (2012).

## **Bordism invariance of indices and the bulk-edge correspondence**

*Shin Hayashi*  
*University of Tokyo*

G. M. Graf and M. Porta defined an index for a two dimensional single-particle tight binding gapped Hamiltonian on the square lattice. This index is defined by the information of bulk (i.e. without Dirichlet boundary condition) and is closely related to edge index. They used this index to show the bulk-edge correspondence.

In this talk, we see that the equality between the 1st Chern number of the Bloch bundle and the Graf-Porta's bulk index can be understood as the bordism invariance of the index and prove the bulk-edge correspondence from the K-theoretic point of view.