Forced Snaking

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We study spatial localization in the real subcritical Ginzburg-Landau equation $u_t = m_0 u + Q(x)u + u_{xx} + d|u|^2 u - |u|^4 u$ with spatially periodic forcing $Q(x)$. When $d > 0$ and $Q \equiv 0$ this equation exhibits bistability between the trivial state $u = 0$ and a homogeneous nontrivial state $u = u_0$ with stationary localized structures which accumulate at the Maxwell point $m_0 = -3d^2/16$. When spatial forcing is included its wavelength is imprinted on u_0 creating conditions favorable to front pinning and hence spatial localization. We use numerical continuation to show that under appropriate conditions such forcing generates a sequence of localized states organized within a snakes-and-ladders structure centered on the Maxwell point, and refer to this phenomenon as *forced snaking*. We determine the stability properties of these states and show that longer length scale forcing leads to stationary trains consisting of a finite number of strongly localized, weakly interacting pulses exhibiting *foliated snaking*.

Morse Decompositions of Regulatory Networks via Determining Nodes

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The regulatory network is a coupled ODE system associated with a network representing regulation relations among variables. This is a mathematical formulation of a biological regulatory network, given by Fiedler et al. (JDDE 2013).

One of the main theorems of their paper is that the global attractor of a regulatory network can be reconstructed if one monitors the information of solutions on the entire negative real line at a well-chosen subset of nodes, called the feedback vertex set (abbrev. FVS). This means that one can understand the nontrivial global dynamics of a regulatory network only from its FVS variables. This is, however, not practical in applications, as one needs to monitor solutions for infinitely long time.

In this talk, we shall show that, if we restrict attention to a coarser information of the dynamics on the global attractor, namely its Morse decomposition, it is sufficient to monitor solutions only for a finite time interval, or even at finitely many time sample points, at a FVS. We shall show a result of numerical computation for Mirsky's circadian rhythm network as a test example.

This is a joint work with Bernold Fiedler (Free University of Berlin, Germany), Gen Kurosawa (RIKEN, Japan), Atsushi Mochizuki (RIKEN, Japan), Hiroe Oka (Ryukoku University, Japan).

Analysis of torus canards and other delayed loss of stability phenomena

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Torus canards have recently been shown to be critical for understanding the transitions between the patterns of time-periodic (or tonic) spiking and bursting of different types. This talk will focus on the analysis of torus canards in systems ranging from the classical forced van der Pol equation, the Hindmarsh-Rose model, and the Morris-Lecar-Terman system, to higher-dimensional systems, such as the Politi-Hofer model, which have two or more slow variables, and in which torus canards are now known to be generic. Along the way, we also discuss a new class of singularities of fast-slow ODEs.

Granular turbulence

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The rheology of dense granular materials strongly depends not only on the shear rate, but also on the fraction of constituent particles. If the fraction is much lower than the jamming point, Bagnold's scaling predicted by kinetic theory well explains the flow behavior. However, if the fraction exceeds the critical value, a finite yield stress is observed in a quasi-static limit as kinetic theory breaks down, where the stress becomes rate-independent of which microscopic origins are still unknown.

Employing molecular dynamics (MD) simulations, we have investigated the microscopic insight into dense granular materials under simple shear deformations, where we observed turbulent-like structures of non-affine velocities (defined as velocity fluctuations around a mean velocity field). From the statistical analyses of non-affine velocities, we found their strong spatial correlations when the system was yielding under quasi-static deformations. In addition, the spectrum of non-affine velocity fields (i.e. energy spectrum) exhibited a clear power-law decay at mesoscale like a usual turbulence. Adopting hydrodynamic equations of yielding granular materials, we derived hydrodynamic modes by perturbation theory and proposed a theoretical expression for the energy spectrum, where a good agreement with the results of MD simulations were confirmed over a wide range of length scales.

Molecular mechanisms leading Kuramoto oscillators

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The Kuramoto model has been widely used to describe the synchronizations of a large set of coupled oscillators. In particular, the Kuramto model successfully captures the key features of synchronization of ~20,000 coupled cellular rhythms in the circadian clocks of our brain. However, due to the abstractness of the Kuramoto model, specific molecular mechanisms underlying the sinusoidal coupling terms have not been identified. In this talk, I will discuss that the combination of intracellular transcriptional repression mechanisms and intercellular coupling mechanisms in the mammalian circadian clocks can lead to the sinusoidal coupling.

Human sperm under a mathematical microscope

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A sperm cell swims with a wave-like deformation of its tail, called a flagellum, to propel in a fluid for conveying genetic information to an egg. From experimental microscopic images of a human sperm, we have theoretically reconstructed the flagellar waveform as a periodic orbit in a low-dimensional PCA phase space. The motion of the swimming sperm is governed by the fluid equations around the cell, and we have numerically computed the Stokes equations of the low Reynolds number flow, using the reconstructed waveform as a boundary condition. The numerical computation based on the boundary element method has successfully reproduced the observed sperm swimming, which enables reliable theoretical predictions for the fertilization phenomena under a microscope. We have then studied flow patterns around the cell with the same waveform, and it is found that the complicated time-dependent flow field can be characterised by a small number of simple flows but of Stokes fundamental solutions.

Distributional enstrophy dissipation caused by singular vortex dynamics in 2D incompressible flow

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It is suggested that the enstrophy dissipation in 2D incompressible flows in the zero viscous limit gives rise to the inertial range corresponding to the forward enstrophy cascade in the energy spectrum of 2D turbulent flows. That is to say, enstrophy dissipating solutions in the 2D incompressible Euler equations might describe 2D turbulence phenomena. However, since smooth solutions in the 2D Euler equations conserve the enstrophy, we need to construct non-smooth solutions that dissipate the enstrophy. In order to accomplish this task, we consider a dispersive regularization of the Euler equations, known as the Euler-alpha equations, for the initial vorticity given by three point vortices and take the alpha zero limit of its global weak solution. We prove that the limit solution converges to a singular self-similar evolution collapsing to a point and then expanding from the point to infinity. We also find that the enstrophy dissipates in the sense of distributions at the collapse time. This indicates that the triple collapse is a generic mechanism for the anomalous enstrophy dissipation in inviscid and incompressible flows. These results are shown not only for alpha regularization but for other kind of regularization such as vortex blob method. The contents are based on the joint works with Takashi Sakajo (Kyoto Univ.).

Slowly moving localized pulse waves in a singularly perturbed two-component bistable system

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We consider the dynamics of a pulse solution formed by two interacting front solutions in a specific a bistable reaction-diffusion system in one dimension. In a singular limit, the dynamics of front positions is replaced by ordinary differential equations (ODEs), whereas that of the continuous medium is given by a partial differential equation. Applying the reduction approach to the resulting resulting mixed ODE-PDE system, we derive the equations of motion for two interacting interfaces, which describe well the pulse behaviors observed in the original PDEs. This is a joint work with Kei Nishi and Yasumasa Nishiura.

Multistate network model for the path-finding problem

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We propose a continuous network model for the path-finding problem. Network nodes are excitable systems of the FitzHugh-Nagumo type that have three equilibrium states depending on input from other nodes. The proposed system spontaneously finds a path connecting two specified nodes, and the path is represented by stable solutions. The system also has a self-recovery property, i.e., the system can find a path when one of the connections in the existing path is suddenly terminated. Further, we demonstrate that the appropriate installation of inhibitory interaction improves the search time.

The Spectral Stability of Bacterial Pulses for Keller-Segel Chemotactic Model

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In this talk I shall talk about our recent work on the stability of a family of traveling waves for a Keller-Segel system with logrithmic chemotactic term, which was first proposed by Keller and Segel in 1971 tomodel the bacteria population chemotaxis in a capillary tube. By applying detailed spectral analysis and Evan's function method, we shall prove the spectral stability/instability of the waves in some appropriate weighted spaces. We shall also talk about our work on the local well-posedness of solution for the Keller-Segel model. It's a joint work with Yi Li (California State University) and Yong Li (Beijing University of Industrial Technology)

Stability of traveling waves for bistable lattice dynamical systems

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In this talk, we will study traveling waves for some systems on a one-dimensional lattice arising in a model of competing species. We will discuss the stability of monotone traveling waves connecting stable equilibria.

Fisher-KPP equation in a time-periodic advective environment with free boundaries

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We consider a reaction-diffusion-advection equation of the form: $u_t = u_{xx} - \beta(t)u_x + f(t,u)$ for $x \in (g(t), h(t))$, where $\beta(t)$ is a time-periodic function and $f(t,u)$ is a Fisher-KPP type of nonlinearity, time-periodic in t, $g(t)$ and $h(t)$ are two free boundaries satisfying Stefan conditions. This equation can be used to describe the population dynamics in time-periodic environment with advection. I will talk about the influence of the advection parameter β on asymptotic behavior of the solutions. It turns out that the influence is qiute different when β is a small, or a medium-sized, or a large function. This talk is based on joint works with H. Gu, N. Sun and M. Zhou.

Resonance tongues appearing in a billiard problem under nonlinear and nonequilibrium conditions

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A four-dimensional dynamical system defined by a system of ordinary differential equations is considered. It was derived by M. Mimura and his collaborators as a mathematical model of self-propelled motions of a camphor disk floating on a water surface. While a camphor disk repeats a uniform motion and reflection as if it is a billiard ball, there are some differences from the mathematical billiards in which a particle exhibits a uniform motion and a completely elastic reflection. A camphor disk changes the direction of motion without collision at the boundary and the reflection is not completely elastic: the angle of incidence is different from that of reflection. Moreover, if the domain is square-shaped, orbits of the mathematical model eventually tends to a limit cycle as time passes. In this study, we consider the system in a rectangular domain. Previous numerical studies have revealed the existence of an attractor which may be periodic, quasi-periodic, or chaotic depending on parameters, e.g. the aspect ratio of the rectangle. We apply theoretical and numerical bifurcation analysis for understanding the bifurcation structure of the system. At first one may think that it results from some bifurcation of the square-shaped limit cycle. We show, however, that this is not the case and the Hopf-Hopf bifurcation of the rest state is the organizing center of the complicated bifurcation structure.

Mathematical modeling and genetic analysis of the proneural wave in the fly brain

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Notch-mediated lateral inhibition regulates binary cell-fate choice, resulting in salt-and-pepper patterns during various developmental processes. However, how Notch signaling behaves in combination with other signaling systems remains elusive. The wave of differentiation in the fly visual center or 'proneural wave' accompanies Notch activity that is propagated without the formation of a salt-and-pepper pattern, implying that Notch does not form a feedback loop of lateral inhibition during this process. However, mathematical modeling and genetic analysis clearly demonstrated that Notch-mediated lateral inhibition is implemented within the proneural wave and that EGF diffusion cancels salt-and-pepper pattern formation. Moreover, the combination of Notch-mediated lateral inhibition and EGF-mediated reaction diffusion enables a novel function of Notch signaling that regulates propagation of the wave of differentiation.

Spontaneous motion using surface tension gradient

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Self-propelled motion has been investigated from the standpoint of physics. Motion of the living things is also a kind of self-propelled motion. We have investigated it using physico-chemical systems, since they have advantages that they are easier to control and their elemental processes are known. In the presentation, I will introduce the motion of an elliptic camphor disk on water and spontaneous motion of a droplet coupled with the pattern formation of Belousov-Zhabotinsky (BZ) reaction. In the former system, the camphor particle moves due to the surface tension gradient at a water surface originating from the profile of the camphor concentration dissolved from the camphor particle. By embedding the asymmetry to the shape of the camphor disk, the motion of the camphor particle is changed due to the system assymmetry, which is analyzed by solving the reaction-diffusion model on the camphor concentration coupled with the dynamics of the position of the camphor particle. In the latter system, the interfacial tension of the droplet is changed coupled with the pattern formation of the BZ reaction inside the droplet. The interfacial tension gradient induces the Marangoni flow inside and outside of the droplet, which drives the droplet itself. The mechanism is discussed based on the hydrodynamics including the exchange of the momentum between the droplet and the outer phase.

The Collective Behaviors of Self-Propelled Particles and Drops through hydrodynamic interactions

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Self-propulsion of particles and drops has attracted lots of attention in the last decades for its potential application to a smart control of biological systems and industrial materials. The system is far away from equilibrium states and the fundamental understandings of such systems are challenging. Recently, several experiments showing spontaneous motion driven by chemical reactions have been proposed and revealed the underlying mechanism of the motion. Accordingly, several simple theoretical models have been extensively studied such as active Brownian particles, squirmers, self-phoretic swimmers and so on. We have theoretically derived a set of nonlinear equations showing a bifurcation between stationary and motile states driven away from an equilibrium state due to chemical reactions. A particular focus is on how hydrodynamic flow destabilizes an isotropic distribution of a concentration field. This symmetry-breaking mechanism of the self-propulsion leads to deformation of a drop; it elongates perpendicular to the direction of motion.

The assemblage of such self-propelled particles gives rise to collective behaviors such as motility-induced phase separation, global polar state, and clustering. The question is how such variety of patterns appear from interactions between the individual elements. Our understandings are still primitive, but I shall try to clarify how hydrodynamic interactions and the interaction mediated by a concentration field lead to the collective behaviors.

Localized Patterns & Spatial Heterogeneities

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We consider the impact of spatial heterogeneities on the dynamics of localized patterns in systems of partial differential equations (in one spatial dimension). We will mostly focus on the most simple possible heterogeneity: a small jump-like defect that appears in models in which some parameters change in value as the spatial variable x crosses through a critical value -- which can be due to natural inhomogeneities, as is typically the case in ecological models, or can be imposed on the model for engineering purposes, as in Josephson junctions. Even such a small, simplified heterogeneity may have a crucial impact on the dynamics of the PDE. We will especially consider the effect of the heterogeneity on the existence of defect solutions, which boils down to finding heteroclinic (or homoclinic) orbits in an n-dimensional dynamical system in `time' x, for which the vector field for $x > 0$ differs slightly from that for $x < 0$ (under the assumption that there is such an orbit in the homogeneous problem). Both the dimension of the problem and the nature of the linearized system near the limit points have a remarkably rich impact on the defect solutions. We complement the general approach by considering two explicit examples: a heterogeneous extended Fisher–Kolmogorov equation ($n = 4$) and a heterogeneous generalized FitzHugh–Nagumo system ($n = 6$).

The Kuramoto model on networks

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Dynamics of a system of large populations of coupled oscillators have been of great interest because collective synchronization phenomena are observed in a variety of areas. The Kuramoto model is often used to investigate such phenomena.

In this talk, the Kuramoto model defined on networks, such as all-to-all coupling, small world network, scale free network, is considered. A bifurcation from the de-synchronous state to the synchronous state will be proved with the aid of the generalized spectral theory.

The dynamics of fibrin gel formation

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Biogels are complex polymeric networks whose proper function is important to many physiological processes. For example, the proper function of mucus gel is important for airway clearance, reproduction, digestion, gastric protection, and disease protection and its failure is involved in cystic fibrosis, gastric ulcers, and reproductive dysfunction. Fibrin clots are crucial for prevention of bleeding after injury but inappropriate formation of clots is implicated in hearts attacks and strokes.

The purpose of this talk is to describe recent advances in the study of the dynamics of fibrin clot formation. In particular, I will derive and discuss features of a new partial differential equation model that describes the growth of fibrin clots as a polymerization/gelation reaction. The solution of this PDE model gives insight into the branching structure of clots that are formed under various physiological conditions.

Pattern Formation in Nonlinear Nonlocal Diffusion Equations

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I will present some results giving the convergence of the spectrum of a family of nonlocal operators to the spectrum of the Laplacian as a parameter approaches zero. From this, with some effort caused by the fact that the nonlocal operator does not have compact resolvent, we can deduce bifurcation in a nonlocal Turing system and in a nonlocal Chafee-Infante problem.

Collective waves in deformable self-propelled particles

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The dynamics of self-propelled objects have been studied extensively after Vicsek et al investigated collective motions introducing a simple stochastic model of alignment mechanism [1]. The transitions between the ordered state for weak noise intensity or high density and the disordered state for strong noise intensity or low density have been investigated by numerical simulations mainly in two dimensions. It has been found that collective waves called traveling bands are spontaneously formed near the transition threshold [2].

We have introduced a model system of deformable self-propelled particles, which has a feature that the propagating velocity of each particle increases as the surrounding density is increased. We have found numerically that traveling waves are formed and that these waves are stable upon head-on collisions [3]. Deformability of particles is the central origin of this soliton-like behavior in contrast to the Vicsek model of point particles, where traveling bands are unstable upon collisions [4].

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Rate of species creation by geographic isolation and recurrent migration.

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We study the time to speciation by geographic isolation for a species living on multiple islands connected by rare migration. We assume that the incompatibility is controlled by many loci, and that individuals differing in loci more than a threshold do not mix genetically with each other. For the case of two populations, we can analyze the system by tracing their genetic distance, defined as the number of incompatibility loci differing between the populations. If each population is nearly monomorphic, the genetic distance follows stochastic processes, which can be analyzed by diffusion equation and by stochastic differential equation (SDE). There exists an intermediate optimal rate of migration that maximises the rate of species creation by recurrent invasion and diversification. We also study the case when the strength of incompatibility gradually increases with the genetic difference. For the case of three or more populations, genetic distances do not form closed stochastic dynamics. To overcome this difficulty, for each locus we define "geographic configuration (GC)" which specifies islands with common alleles, and we trace the stochastic transitions between different geographic configurations by SDEs for the number of loci with different GC. We also discuss how the speciation rate changes with geographical structure.

Waves in smoldering combustion under micro-gravity

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Combustion is a fast oxidation process and exhibits diverse behavior according to experimental conditions. On the other hand, when there is no natural convection, of gas such as experiments in the space shuttle. It is observed under this situation, that unexpected smoldering combustion develops. IN this talk, we discuss waves arising in this combustion by using a macro-scopic PDE model.

Interaction of gliomas with their micro-environment: A mathematical model

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Malignant gliomas are the most common type of brain cancer, which arise from glial cells, and in their most aggressive form are called GBMs. GBMs are highly invasive and difficult to treat because cells migrate into surrounding healthy brain tissue rapidly, and thus these tumors are difficult to completely remove surgically. GIMs, which can comprise up to one third of the total tumor mass (Markovic et al, 2009), are present in both intact glioma tissue and necrotic areas. They apparently originate from both resident brain macrophages (microglia) and newly recruited monocyte-derived macrophages from the circulation. Activated GIMs exhibit several phenotypes: one called M1 for classically activated, tumor suppressive, and another called M2 for alternatively activated, tumor promoting, and immunosuppressive (Mantovani et al, 2013). Within a tumor the balance between these phenotypes is typically shifted to the M2 form (Pollard, 2009). Numerous factors secreted by glioma cells can influence GIM recruitment and phenotypic switching, including growth factors, chemokines, cytokines and matrix proteins (Coniglio et al, 2012; Wang et al, 2012). In this work, we focus on mutual interaction between a glioma and M1/M2 microglia mediated by CSF-1, TGFbeta, and EGF. Up-regulated TGFbeta leads to up-regulation of Smad within the tumor cells and secretion of MMPs, leading to proteolysis for EMT process and cell infiltration. The mathematical model consists of densities of glioma cells, M1 type cells, M2 type cells, and concentrations of CSF-1, EGF, TGFbeta, Extracellular matrix, and MMPs. We developed the model to investigate the mutual interactions between tumor cells in the upper chamber and microglia in the lower chamber. In the experiments, Boyden invasion assay was used to show that this mutual interaction induces glioma infiltration in vitro and in vivo. We show that our simulation results are in good agreement with the experimental data and we generate several hypotheses that should be tested in future experiments in vivo.

For the second part of the talk, we investigate the role of microenvironment in glioma cell infiltration, especially, in the context of glioma stem cells. We focus on the signaling networks within a glioma cell that control the EMT, therefore, increasing further growth and tumorigenicity.

Interplay between bistability and finite wavenumber Hopf: Actin and molecular motor waves

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Nonlinear chemical based waves, e.g., the Belousov-Zhabotinsky reaction, set also the stage for biological mechanisms, including action potentials, intracellular calcium, and slime mold waves. The phenomenology can be often devised via a simple two variable reaction-diffusion models by two universality classes: Hopf oscillations and excitability. However, examination of intracellular actin and molecular motor waves reveals that the above classifications do not apply to these systems. Instead, a novel approach that combines bistability and finite wavenumber is suggested to tackle the complex spatiotemporal mechanisms and the pattern selection. Using at least three variable reaction-diffusion generic models, bifurcation analysis and numerical simulations, we find both a good agreement with experimental observations and distinct dynamical systems realizations.

Sizes of patterns that are associated with leaf and blood vessel formation

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I will discuss two topics related to pattern size, which was changed based on temperature and aging.

The first topic is the effect of pattern size on leaf diversity. North American lake cress, Rorippa aquatica, has heterophyllous leaves depending on the temperature that they have been exposed to, showing various peripheral complexities in leaves from simple elliptical to highly branched shapes. This temperature-dependent leaf variation was considered to be associated with the size of the biased deformation that exists across the blade of the leaf from the proximal to the distal end. The size of the pattern involved in this biased deformation is expected to decrease with increasing temperature, as observed in a reaction-diffusion system. This hypothesis was confirmed by conducting a series of temperature-shift experiments and simulations.

The second topic is regarding endothelial cells, which cover the lumen of blood vessels. They are capable of forming self-organized patterns in vivo. The sizes of these patterns were different based on the passages number of the endothelial cells. Endothelial cell aging is known to increase apoptosis and change chemotactic movements. Therefore, we analyzed these effects on pattern size. A simple partial differential equation (PDE) model was constructed and linear stability analysis was performed to obtain the change of most unstable wavelength.

Instability caused by non-local interaction and its reaction-diffusion approximation

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Recently, various non-local interactions which influence globally in space arise in many fields, for example, in the spatial dispersals of the living things, the neural firing phenomenon in brain and the pigment cells in the skin of the zebra fish. To reveal the mechanisms of the pattern formations many mathematical models with convolutions were proposed. We found from numerical simulations that the destabilization of the solution was very sensitive for the shape of the convolution kernel. To specify this relationship, we approximate the non-local interaction by reaction-diffusion systems (RDS) using the singular limit analysis. Through this approximation we reveal not only the relationship between the destabilization and the kernel shape but also that the destabilization is the diffusion driven instability. Finally, it is shown that any non-local interactions with the even convolution kernel can be realized by the RDS by increasing the components.

Evolution and Interaction of Wave Packets in Dispersive Systems

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The analysis of localized structures is an important method for gaining insight into the rich dynamics of nonlinear partial differential equations. While localized structures such as front solutions or pulses are well-understood, the theory for wave packets is still developing. This talk presents some new results for the evolution and interaction of wave packets in the context of several different dispersive equations, with particular emphasis on nonlinear wave equations with periodic coefficients.

Localized Bioconvection Patterns: Experiments and Theories

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The topic of this talk is the spatially localized bioconvection patterns when a suspension of Euglena gracilis that is a unicellular flagellated photosynthetic alga is illuminated below. The hierarchical structure of the patterns from micrometer scale to centimeter scale is discussed in terms of individual motions, cell density flux of a suspension, and hydrodynamical model based on the experimental results.

Mechanism of Spontaneous Spiral Formation on Heterogeneous Excitable Media

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Spontaneous spiral formation has been observed on heterogeneous excitable media without any interaction between waves, although the widely believed origin of spiral waves is the interaction of traveling waves on homogeneous media. We suggest one possible origin of spiral waves, which is a unidirectional site stochastically generated by a heterogeneity of the reaction field, using a Belousov-Zhabotinsky reaction and its mathematical model. To clarify the cause of unidirectional behavior, we focused on the inhibitor concentration around the path and found a spatial asymmetric profile. Furthermore, we found that the spiral wave vanished due to a slight reduction of the excitability. These results reveal a gentle approach for controlling the appearance of a spiral wave on an excitable medium.

Nonlinear Dynamics, High Dimensional Data, and Persistent Homology

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It is almost cliche at this point to note that high dimensional data is being collected from experiments or generated through numerical simulation at an unprecedented rate and that this rate will continue rising extremely rapidly for the foreseeable future. Our interest is in data associated with high dimensional nonlinear complex spatiotemporal dynamics. The focus of this talk is on our efforts to use persistent homology as a dimension reduction technique so that the analysis of the nonlinear dynamics can be done in the space of persistence diagrams. I will present some results associated with dynamics of fluid convection and dense granular media and will try to highlight open questions.

Kernel method on persistence diagram

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In topological data analysis(TDA), persistence diagrams are widely used to describe the robust and noisy topological properties in data. In this talk, I will propose a kernel method on persistence diagrams to develop a statistical framework in TDA. Then, a new distance on persistence diagrams is defined by our kernel and has a stability property. The main contribution of our method is to control the effect of persistence flexibly. As an application, we show that our method can clearly detect the glass transition temperature in SiO2.

Tracking errors in the space of persistence diagrams

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Topological data analysis is a growing branch of mathematics that concerns the study of the shape of inherently high-dimensional data. One particularly useful tool is that of persistent homology, where n-dimensional topological features of the data are encoded into a collection of coordinates in the plane, called a persistence diagram. For particularly large data sets or noisy data, approximations or smoothing must first be applied to either make computations possible or to clean the signature of the features. In this talk, we will introduce a recent result that makes it possible to compute rigorous bounds on the errors introduced by such approximations. We will primarily focus on two examples that come from the study of the fluid dynamics of Rayleigh-Bénard convection flow: estimating the shape of a large point cloud, and studying features in noisy image data.

Time Scales in the Dynamics of Granular Materials

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In this talk we will introduce the methods of topological data analysis. Namely, the persistence diagrams which are a relatively new topological tool for describing and quantifying complicated patterns in a simple but meaningful way. We will demonstrate this technique on patterns appearing in dense granular media. This procedure allows us to transform experimental or numerical data, from experiment or simulation, into a point cloud in the space of persistence diagrams. There are a variety of metrics that can be imposed on the space of persistence diagrams. By choosing different metrics one can interrogate the pattern locally or globally, which provides deeper insight into the dynamics of the process of pattern formation. We will use these metrics to identify the important time scales at which behavior of the system changes. We will also discuss a physical interpretation of these time scales.