Simplicial networks and effective resistance

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We introduce the notion of effective resistance for a simplicial network \((X,R)\) where \(X\) is a simplicial complex and \(R\) is a set of resistances for the top simplices, and prove two formulas generalizing previous results concerning effective resistance for resistor networks. Our approach, based on combinatorial Hodge theory, is to assign a unique harmonic class to a current generator \(\sigma\), an extra top-dimensional simplex to be attached to \(X\). We will show that the harmonic class gives rise to the current \(I_\sigma\) and the voltage \(V_\sigma\) for \(X \cup \sigma\), satisfying Thompson’s energy-minimizing principle and Ohm’s law for simplicial networks.

The effective resistance \(R_\sigma\) of a current generator \(\sigma\) shall be defined as a ratio of the \(\sigma\)-components of \(V_\sigma\) and \(I_\sigma\). By introducing potential for voltage vectors, we present a formula for \(R_\sigma\) via the inverse of the weighted combinatorial Laplacian of \(X\) in codimension one. We also derive a formula for \(R_\sigma\) via weighted high-dimensional tree-numbers for \(X\), providing a combinatorial interpretation for \(R_\sigma\). As an application, we generalize Foster’s Theorem, and discuss various high-dimensional examples. Moreover, as a tool for analyzing simplicial networks, we suggest a definition of information centrality for simplicial networks. This is a joint work with Woong Kook.