Homology-Based Immersions of Metric Objects

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Consider the surface of a hand naturally embedded in 3D space, and pick a point, say at the tip of the index finger. By considering intrinsic neighborhoods of increasing radius around this basepoint, and computing the associated persistent homology, one can extract a stable multi-scale point signature that encodes both global connectivity and local geometry. This construction was considered in a 2015 paper of Carriere, Oudot, and Ovsjanikov, and naturally leads to the question: "how does one pick the correct point? Surely some points give more information than others." We can deflect this question by refusing to choose a single point, instead allowing each point in our space to serve as a basepoint in its turn. What we obtain is an immersion of our metric object as a subset of barcode space. As a variant of this procedure, we can also start with a metric-measure object and subsequently obtain a probability distribution on barcode space. In the case of metric graphs, we show stability for these constructions in the appropriate metrics. It is a matter of ongoing research to determine how discriminative these constructions are and how they might be generalized to higher-dimensional spaces. This is joint work with Steve Oudot.

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