Reeb Graph Smoothing via Cosheaves

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Abstract. Topological persistence is perhaps the most well-known aspect of applied algebraic topology. In this talk, I will give an introduction to the category-theoretic perspective that unifies several different constructions: persistence modules, merge trees, Reeb graphs. The case of Reeb graphs is particularly fruitful: these are ordinarily thought of as topological graphs equipped with a non-degenerate real-valued function, but one can also interpret them as cosheaves over the real line.

Whereas sheaves are widely used in algebraic geometry, the dual theory of cosheaves is perhaps less commonly exploited. Following [de Silva, Munch, Patel (2016)], I will explain the correspondence between Reeb graphs and Reeb cosheaves, and use it to define a metric on the class of all Reeb graphs, as well as a 1parameter semigroup of 'smoothing' operators that progressively simplify the topology of a given Reeb graph. Finally, I will present an algorithm [de Silva, Smirnov, Yu (2017, unpublished)] that produces, in a single calculation, the description of the smoothings of a given Reeb graph at all possible parameter values.