Magnitude homology

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Magnitude homology is a homology theory of enriched categories, proposed by Michael Shulman late last year. It therefore specializes to (among other things) a homology theory of ordinary categories and a homology theory of metric spaces. This metric homology theory is something new, and is different from persistent homology. As a sample result, the first homology of a subset X of Euclidean space detects whether X is convex.

Like all homology theories, magnitude homology has an Euler characteristic, defined as the alternating sum of the ranks of the homology groups. Often this sum diverges, so we have to use some formal trickery to evaluate it. In this way, we end up with an Euler characteristic that is often not an integer. This number is called the *magnitude* of the enriched category. In the setting of compact metric spaces, magnitude is closely related to volume, surface area, curvature, and other classical invariants of geometry. In the special setting of finite metric spaces, magnitude appears to provide information of interest about point-sets, such as the apparent dimension and number of clusters at different length scales.

I will give an overview of these developments, assuming little categorical knowledge.

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