The Rise and Spread of Algebraic Topology

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As algebraic topology becomes more important in applied mathematics it is worth looking back to see how this subject has changed our outlook on mathematics in general. When Noether moved from working with Betti numbers to homology groups, she forced a new outlook on topological invariants: namely, they are often *functors*, with two invariants counting as "the same" if they are *naturally isomorphic*. To formalize this it was necessary to invent categories, and to formalize the analogy between natural isomorphisms between functors and homotopies between maps it was necessary to invent 2-categories. These are just the first steps in the "homotopification" of mathematics, a trend in which algebra more and more comes to resemble topology, and ultimately abstract "spaces" (for example, homotopy types) are considered as fundamental as sets. It is natural to wonder whether topological data analysis is a step in the spread of these ideas into applied mathematics, and how the importance of "robustness" in applications will influence algebraic topology.

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