【CL8-2】

# The law of large number of the lifetime in random complex processes

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We consider a higher dimensional generalization of Frieze's ζ(3)-limit theorem. Frieze's theorem states that the expected weight of the minimum spanning tree in the complete graph, whose edges are all independently weighted by uniform random variables over the interval [0, 1], converges to ζ(3) as the number of vertices goes to infinity. By using the terminology of persistent homology, Frieze's theorem can be regarded as the limiting theorem of the expected value of the 0-th lifetime sum in an increasing family of the Erdös—Rényi random graphs, which is called by the Erdös—Rényi process. At this point of view, Y. Hiraoka and T. Shirai studied the behavior of the expected lifetime sums in the Linial—Meshulam complex process and the clique complex process, which are higher dimensional generalizations of the Erdös—Rényi process [1].

we first establish a generalization of the cohomology vanishing theorem based on the quantitative approach. By using this estimate, we improve the upper bound of the order of the expected lifetime sum in the clique complex process, which was obtained in [1]. Especially, we determine the exact order of the expected lifetime sum in the clique complex process. Second, using the same estimate, we prove the law of large number of the lifetime in the (k+1)-Linial—Meshulam complex process with n vertices. Specifically, we prove the following three statements. The last two statements can be regarded as a higher dimensional generalization of Frieze's theorem.

1. The k-th expected lifetime sum $L_k$ in the (k+1)-Linial—Meshulam complex process behaves in $O(n^k)$.
2. The expected value of the random variable $L_k/n^k$ converges to a positive constant $I_k$ as n goes to infinity.
3. The random variable $L_k/n^k$ converges in probability to the constant $I_k$.

We remark that the first statement was proved in [1] by other method and the second statement was conjectured from a formal discussion.

[1] Y. Hiraoka and T. Shirai. Minimum spanning acycle and lifetime of persistent homology in the Linial—Meshulam process. To appear in Random Structures and Algorithms.

*This is joint work with Masanori Hino.*