【CL7-2】

Simplicial networks and effective resistance

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We introduce the notion of effective resistance for a *simplicial network* (X,R) where X is a simplicial complex and R is a set of resistances for the top simplices, and prove two formulas generalizing previous results concerning effective resistance for resistor networks. Our approach, based on combinatorial Hodge theory, is to assign a unique harmonic class to a *current generator* σ, an extra top-dimensional simplex to be attached to X. We will show that the harmonic class gives rise to the *current* I_σ and the *voltage* V_σ for $X \cup \sigma$, satisfying Thompson's energy-minimizing principle and Ohm's law for simplicial networks.

The effective resistance R_σ of a current generator σ shall be defined as a ratio of the σ-components of V_σ and I_σ. By introducing *potential* for voltage vectors, we present a formula for R_σ via the inverse of the weighted combinatorial Laplacian of X in codimension one. We also derive a formula for R_σ via weighted high-dimensional tree-numbers for X, providing a combinatorial interpretation for R_σ. As an application, we generalize Foster's Theorem, and discuss various high-dimensional examples. Moreover, as a tool for analyzing simplicial networks, we suggest a definition of information centrality for simplicial networks. This is a joint work with Woong Kook.