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A Discrete Morse Theoretic Approach for Computing Connection Matrices

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Algebraic topology and dynamical systems are intimately related: the algebra may constrain or force the existence of certain dynamics. Morse homology is the prototypical theory grounded in this observation. Conley theory is a far-reaching topological generalization of Morse theory and a great deal of effort over the last few decades has established a computational version of the Conley theory. The computational Conley theory is a marriage of combinatorics, order theory and algebraic topology, and has proven effective in tackling dynamical problems.

Within the Conley theory the connection matrix is the mathematical object which transforms the approach into a truly homological theory; it is the Conley-theoretic generalization of the Morse boundary operator. We discuss how the connection matrix can be computed efficiently with discrete Morse theoretic techniques. We will introduce a software package for such computations. Finally, we demonstrate our techniques with an application of our theory and software to the setting of a Morse theory on spaces of braid diagrams introduced by R. Vandervorst et al.

This application allows us to prove forcing theorems for stationary and periodic solutions and connecting orbits in a class of parabolic PDEs.